Statistical Modeling of the Patches DC Component for Low-Frequency Noise Reduction

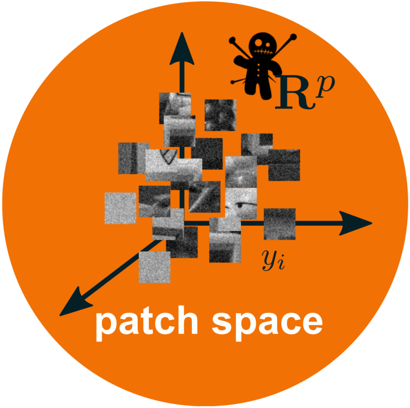
Antoine Houdard
IMB, Université de Bordeaux

27th European Signal Processing Conference

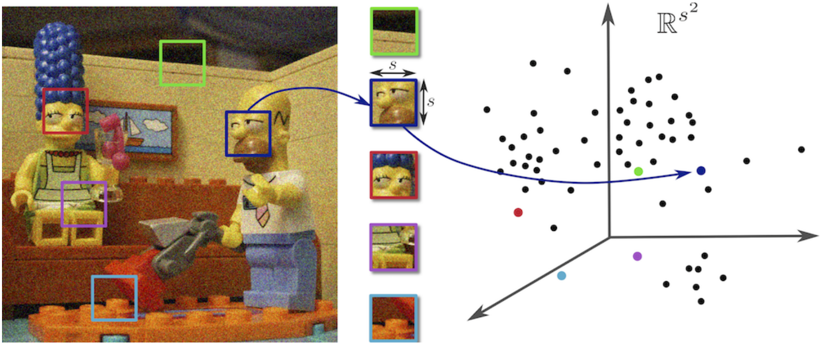
A Coroña, September 2–6, 2019

Context

patch-based image denoising



1. Patch extraction



observed patches $\{y_1, \ldots, y_n\}$

2. Put a prior model on the clean patches

$$Y_i = X_i + N_i$$

2. Put a prior model on the clean patches

white gaussian noise

$$\mathcal{N}(0, \sigma^2 \mathrm{I}_p)$$

$$Y_i = X_i + N_i$$

2. Put a prior model on the clean patches

white gaussian noise

$$\mathcal{N}(0, \sigma^2 \mathrm{I}_p)$$

$$Y_i = X_i + N_i$$

clean patches

$$\pi(x; \theta)$$

2. Put a prior model on the clean patches

observed patches

$\phi(y;\theta)$

white gaussian noise

$\mathcal{N}(0, \sigma^2 \mathrm{I}_p)$

$$Y_i = X_i + N_i$$

clean patches

$\pi(x;\theta)$

3. inference of the parameters θ

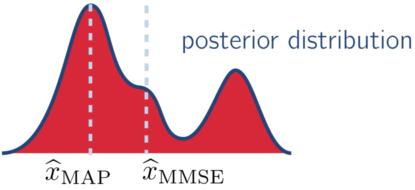
$$\mathcal{L}(y, \theta) = -\sum_i \log\left(\phi(y_i; \theta)\right)$$

3. inference of the parameters θ

$$\mathcal{L}(y, \theta) = -\sum_i \log\left(\phi(y_i; \theta)\right)$$

4. clean patches estimation

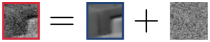
$$\hat{x}_{\mathsf{MMSE}_i} = \mathbf{E}[X_i | Y_i = y_i]$$



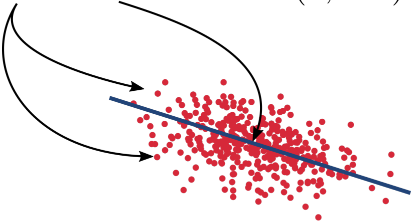
in the literature

- ⋆ Patch-based PCA [Deledalle, Salmon, Dalalyan, 2011]
- ⋆ EPLL [Zora, Weiss, 2011]
- ⋆ NL-Bayes [Lebrun, Buades, Morel 2012]
- ⋆ SURE Guided Gaussian Mixture Image Denoising [Wang, Morel, 2013]
- ⋆ Single-frame image denoising using gaussian mixtures [Teodoro, Almeida, Figueiredo, 2015]
- ⋆ HDMI [H., Bouveyron, Delon, 2018]
- ⋆ ...

This work: HDMI enhancement

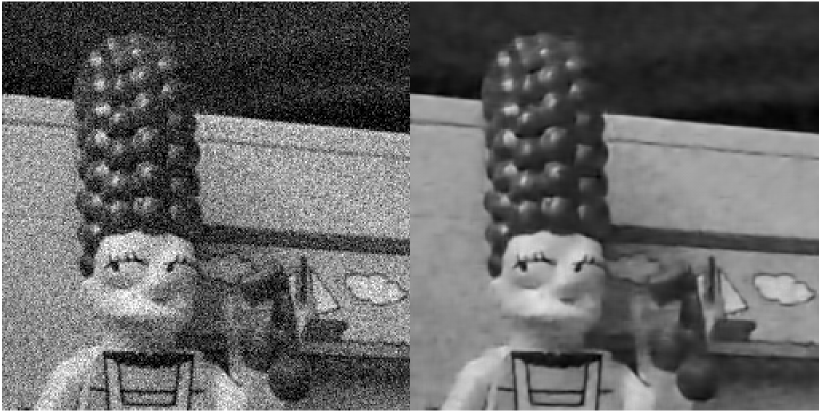


$$Y = X + N \sim \mathcal{N}(0, \sigma^2 \mathbf{I})$$



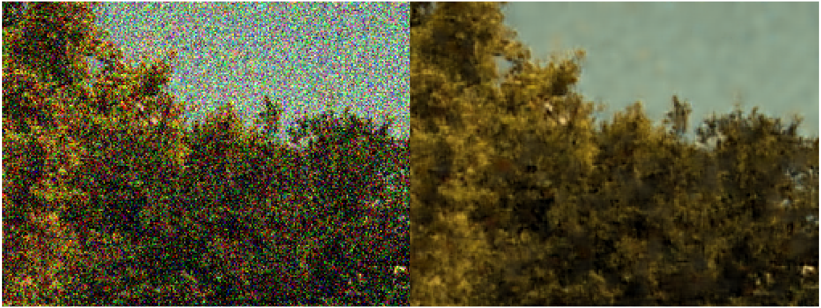
1. Modeling of the patches distribution with a GMM with dimension reduction
2. Inference of the parameters with EM algorithm
3. Estimation of the clean patches with conditional expectation

Good for textures but low-frequency residual noise



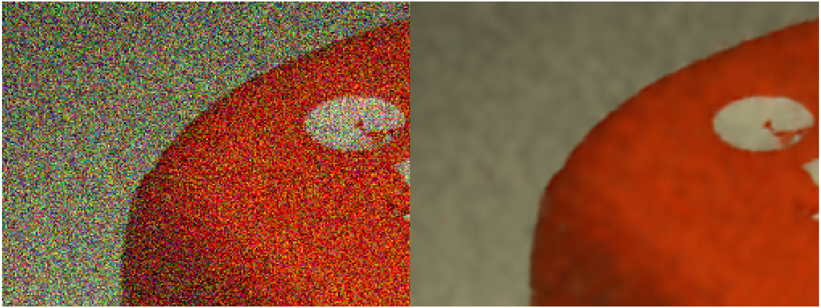
noise 20% & patches 7×7

Good for textures but low-frequency residual noise



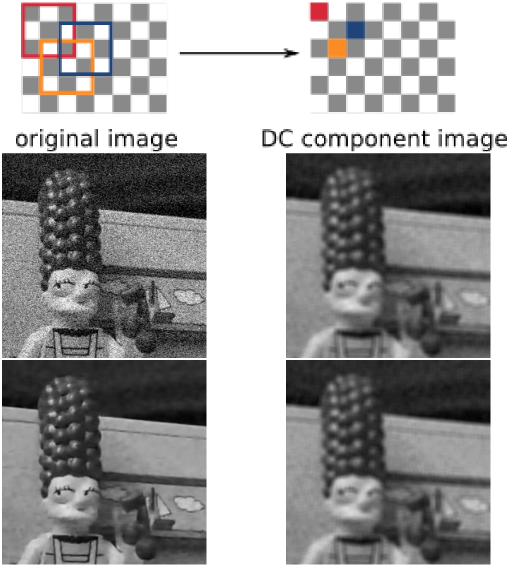
noise 20% & patches 7×7

Good for textures but low-frequency residual noise



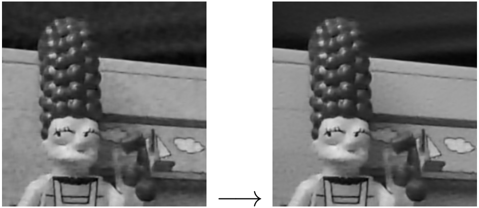
noise 20% & patches 7×7

Observation: this comes from the patches DC component



Idea: denoise the DC component image separately

the denoising can therefore be enhanced



Noise modeling on the DC image

- the DC component $\overline{Y}_i = \frac{1}{p}\sum_{j=1}^{p} Y_i(j)$ correspond to the pixel i of the DC image
- noise model on the DC component

$$\overline{Y}_i = \overline{X}_i + \overline{N}_i \in \mathbf{R},$$

with $\overline{N}_i$ not independant Gaussian random variables

Noise modeling on the DC image

extraction of patches from the DC image

$$Z_i = W_i + M_i,$$

where $Z_i = \pi_i(\overline{Y})$, $W_i = \pi_i(\overline{X})$ and $M_i = \pi_i(\overline{N})$.

Proposition

$M_i \sim \mathcal{N}(0_p, \Sigma_{M_i})$ with

$$\Sigma_{M_i} = \frac{\sigma^2}{p^2} B \otimes B,$$

where

$$B = \begin{pmatrix} s & (s-1) & \cdots & 1 \\ (s-1) & s & \ddots & \vdots \\ \vdots & \ddots & \ddots & (s-1) \\ 1 & \cdots & (s-1) & s \end{pmatrix},$$

Noise whitening

Proposition

Σ_{M_i} is symmetric positive-definite and there exists L invertible such that $B \otimes B = LL^T$.

Noise whitening

Proposition

Σ_{M_i} is symmetric positive-definite and there exists L invertible such that $B \otimes B = LL^T$.

$$L^{-1}Z_i = L^{-1}W_i + L^{-1}M_i,$$

denising problem with white Gaussian noise of variance σ^2/p^2.

let $f_{denoise}$ be a patch denoiser (ex. HDMI), an estimate of W_i is

$$\widehat{W_i} = L\, f_{denoise}\left(L^{-1}W_i\right)$$

afterwards DC correction

1. estimate $\widehat{X}_i = f_{denoise}(Y_i)$ for each patch
2. estimate $\widehat{\overline{X_i}}$ by denoising the DC image
3. replace the DC component of each patch with this estimate

$$h\left(X_i\right) = \widehat{X_i} - \overline{\widehat{X_i}}\mathbf{1}_p + \widehat{\overline{X_i}}\mathbf{1}_p.$$

Results

noisy image

HDMI denoised

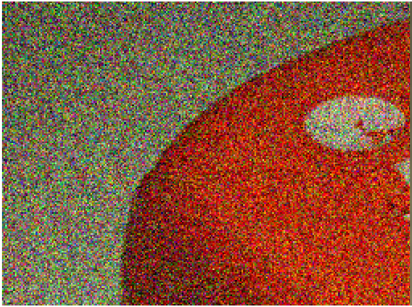
Results

noisy image

HDMI denoised DC corrected

Results

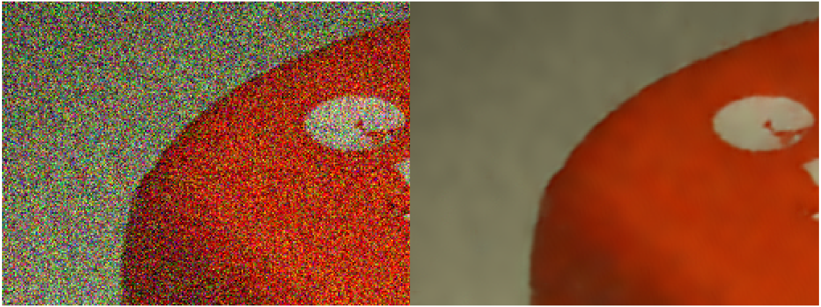
noisy image



HDMI denoised

Results

noisy image HDMI denoised DC corrected



Results

noisy image

Results

HDMI denoised

Results

HDMI denoised DC corrected

Results

noisy image

Results

HDMI denoised

Results

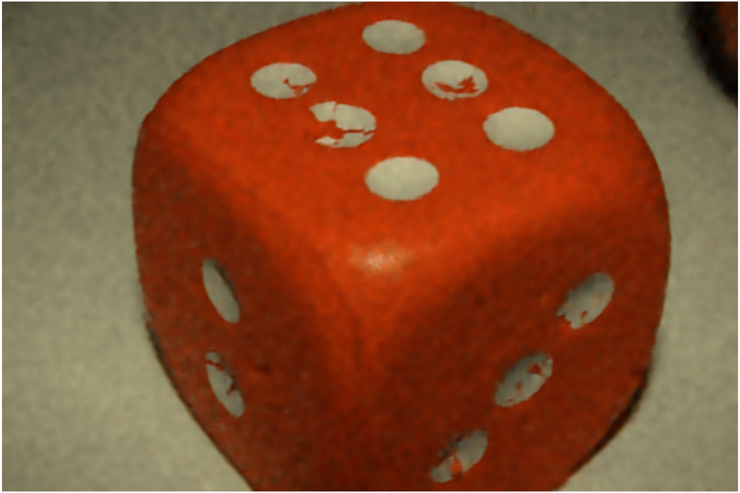
HDMI denoised DC corrected

Results

noisy image

Results

HDMI denoised



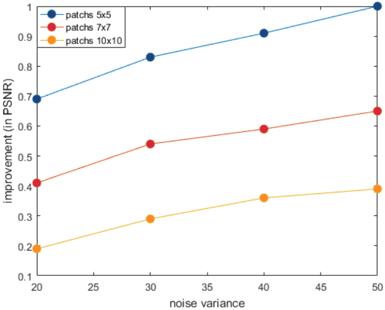
Results

HDMI denoised DC corrected

Concluding remarks

We proposed to denoise the DC component of the patches that

- ⋆ can be rewritten as an additive white Gaussian noise problem
- ⋆ improves the denoising result both visually and qualitatively



- ⋆ can be used with any patch denoising method $f_{denoise}$

further work link with multi-scale frameworks and generalization

Thank you for your attention!

More information on the HDMI method and this work on:
houdard.wp.imt.fr